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To: Dr. D. Mills
Director of Research

SUMMARY OF RESEARCH ACTIVITIES OF A.M. LEITE DA SILVA AT ONTARIO HYDRO

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This report summarizes all the research activities that I have been involved with in the Operations Research (OR) Department during my sabbatical leave from the Electrical Engineering Department, Catholic University of Rio de Janeiro, Brazil. This covers the period from March 19, 1990, until September 27, 1991. Most of the research activities were concerned with the application of probabilistic methods in electric power systems. A list of publications is provided, as well as an Appendix containing a major report completed with the cooperation of Drs. J. Endrenyi and L. Wang from the OR Department.

1. INTRODUCTION

The reliability of an electric power system is, and always has been, a prime concern of utility management and engineers. In many parts of the world, the public has grown accustomed to a service of extremely high quality and is unwilling to lower the existing standards to any significant degree. While a growing number of constraints are placed on the planning and operation of electric power systems, utilities are obliged to maintain the customary reliability of their service.

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Reliability of bulk power systems (generation plus high-voltage transmission) is considered an umbrella concept with no quantitative interpretation. It is broken down into two attributes, both still umbrella concepts: adequacy and security. These are defined as follows: *Adequacy* is the ability of the system to supply the aggregate electric power and energy requirements of the customers, within component ratings and voltage limits, taking into account planned and unplanned component outages. *Security* is the ability of the system to withstand sudden disturbances such as the unanticipated losses of system components. Clearly, security assessments involve transient stability evaluations, while adequacy assessments concentrate on the steady state conditions after a disturbance or incident, such as component failure. Undoubtedly, both aspects are fundamental when evaluating the performance of power systems under uncertainties.

In 1989, with my sabbatical year approaching, I contacted Dr. J. Endrenyi together with other professional colleagues about joint work during the sabbatical period. Based on a number of considerations, I decided to choose Dr. Endrenyi's proposal of spending the year at Ontario Hydro's Research Division. By the end of March, 1990, I started my work in Ontario Hydro, being financially assisted by the Catholic University of Rio de Janeiro (PUC-RJ), and also by the Conselho Nacional de Desenvolvimento Cientifico e Tecnologico (CNPq) of the Secretary for Science and Technology of Brazil. When the sabbatical period was extended from April 2, 1991, until September 27, 1991, for this six-month period I was assisted by Ontario Hydro.

At the outset, different topics of research were discussed with Dr. J. Endrenyi. We agreed that the security assessment of bulk power systems deserved our primary attention. On the other hand, at that time, I still had under my supervision 1 MSc and 4 PhD students, most of them working on the final phase of their theses, discussing topics in the area of adequacy assessment. Therefore, the research activities that I was involved with in the OR Department included both adequacy and security assessments (as well as other topics). In the next sections of this report, these research activities will be summarized.

2. RESEARCH ACTIVITIES

2.1 Adequacy Assessment

There are two basic adequacy techniques for assessing bulk system reliability indices: probabilistic load flow (PLF) and composite reliability evaluation (CRE). Adequacy indices evaluated before using *remedial actions* for resolving network problems can be obtained by a PLF algorithm. These include: probability of any particular power flow being greater than the corresponding equipment thermal rating; probability of a busbar voltage being outside acceptable levels, etc. Such indices measure the adequacy of the available generation and transmission capacities as well as the effectiveness of dispatching policy (particularly in hydrothermal power systems) at various load levels.

Adequacy indices evaluated after using remedial actions can be obtained by CRE algorithms. These include: loss of load probability, expected energy not supplied, etc. The difference between the probability of system problems before and after the use of remedial actions reflects the

benefit of these remedial actions, i.e., it measures the intrinsic flexibility of the system. Therefore, these two set of indices indicate, respectively, the "pessimistic" and "optimistic" levels of reliability in the system.

In some utilities the establishment of remedial action algorithms has been a source of conflict between planning and operating staff: planning tend to see CRE indices as relative measures of system reliability and therefore useful for comparing expansion alternatives; operators would rather have CRE indices as absolute measures of system reliability, i.e. reflecting what is actually happening to the customers. However, it should be recognized that both PLF and CRE are similar and useful tools for identifying system weakness.

PLF Analysis: Before coming to Toronto, with the cooperation of my graduate students and some engineers from ELETROBRAS, "Centrais Eletricas Brasileiras", we have started many studies in this area of research, and most of them [1-5] were completed during 1990. I have presented 3 papers [2-4] at the Third International Conference on Probabilistic Methods Applied to Electric Power Systems in London, UK, in July, 1991, and another one [5] at the CIGRE Symposium on Electric Power Systems Reliability (EPSR) in Montreal, September, 1991.

CRE Studies: In this area, most of the work were carried out in cooperation with some researchers from CEPEL, "Centro de Pesquisas de Energia Eletrica" (Electric Power Research Centre) in Brazil. We have shown how *frequency and duration* indices can be calculated using nonsequencial Monte Carlo simulation methods [6-7], and also we have proposed some computational improvements for analytical methods [8]. I participated at the 1991 IEEE-PES Summer Meeting in San Diego, USA, at the end of July, not only sharing the presentation of paper [6] with Dr. A.C.G. Melo from CEPEL, but also as the Chairman of the session "Reliability Techniques - Equipment and System Analysis". For this meeting, I was fully supported by Ontario Hydro. I presented paper [8] at the CIGRE Symposium on EPSR in Montreal.

Hydrothermal Generating Systems: We have proposed a general and efficient methodology based on rounded discrete convolution to evaluate frequency and duration indices for large hydrothermal generating systems [9]. This work was extended in order to model parameter uncertainties [10].

2.2 Security Assessment

Reviews of previous work in this area were carried out focusing on the solution approaches and indices of system security. Although a large number of papers have been published, there was a lack of appropriate modelling techniques and of proper linkage to power system adequacy studies. The need for new models and indices, based on probabilistic concepts, warranted further investigation. New Markovian models were developed to deal with the different types of disturbances and the actions initiated by the protection system. A prototype security program was

also developed in order to test and validate these new models. This program was applied to a small sample system to demonstrate the proposed concepts. All these new achievements were compiled in a draft report [11], followed by another one [12], where the possibility of using sequential Monte Carlo simulation was discussed.

After discussions with Dr. J. Endrenyi and also Dr. L. Wang, and using a set of draft notes by J. Endrenyi, terms and definitions were arrived at, to give a better shape and consistency to the previous report [11]. The final work, representing the first stage in the development of a probabilistic adequacy/security methodology, can be summarized as follows. An integrated approach for adequacy and security evaluation of bulk power systems has been proposed. This new approach has led to the development of a new set of probabilistic indices to measure the static and dynamic losses of load, and to assess the first impact after the occurrence of a disturbance. The method was applied to a sample system and the effects of variations of selected parameters on the proposed reliability indices were studied.

A paper [13] containing part of this work was presented at the CIGRE Symposium on EPSR in Montreal and the complete work [14] was submitted to the 1992 IEEE-PES Winter Meeting in New York. A copy of the latter paper is attached as an Appendix to this report.

2.3 Reliability - General Concepts

The concept of *First Passage Times* [1] has been reviewed and generalized for systems with any number of states. It has been demonstrated how the standard deviation associated with these times can be evaluated. Moreover, the theoretical distribution was calculated and an algorithm was proposed to evaluate these distributions for practical systems. All these developments are described in a draft report [15], written in August, 1991. Several discussions were carried out with Drs. J. Endrenyi, G.J. Anders and Stephen Lane-Smith.

I also dedicated part of my time in 1990, to reading the draft of some sections of "Reliability Techniques in Large Electric Power Systems" [2], authored by L. Wang and J. Endrenyi.

2.4 Other Research Activities

I have also been involved in the following topics of research.

Dynamic Equivalents: This work was part of a project with ELETROBRAS, Brazil, and basically deals with the identification and subsequent computation of coherency-based dynamic equivalents

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[1] G.J. Anders, *Probability Concepts in Electric Power Systems*, John Wiley & Sons, New York, 1990.

[2] A Chapter in *Control and Dynamic Systems*, Edited by C.T. Leondes, Academic Press, Inc., 1991.

for transient stability studies. It has recently been accepted for publication in the IEE Proceedings Part C [16].

Power System Education in Brazil: The aim of this work [17], which started in 1989, was to present the main feature of power engineering education in Brazil and analyze the current shortage of power students.

3. CONCLUSIONS & ACKNOWLEDGEMENTS

This report presents a brief summary of my research activities in the Operations Research (OR) Department during my sabbatical leave from the Electrical Engineering (EE) Department, PUC-RJ, covering the period from March, 1990, to September, 1991. It was a fruitful experience which should continue through future cooperation between our two institutions. The following topics should receive our attention:

- The extension of the conventional composite adequacy programs to include the concepts and models proposed in [13-14].
- The application of the extended program to real-sized power systems.
- The use of direct or energy function methods to reduce the computational costs associated with the security evaluations.
- Improvements in the modelling of cascading events and restoration processes.
- The evaluation of the remaining life of the stator isolation of hydraulic generating units based on some concepts proposed in [15].

In Brazil, I would like to thank my colleagues from the EE Dept., Catholic University of Rio de Janeiro, and also the assistance granted by CNPq and FINEP. In Canada, special thanks goes to Dr. J. Endrenyi for his full support and friendship, to Dr. L. Wang, Head of the Probability and Reliability Unit, and also to all my colleagues from the OR Dept. , Ontario Hydro Research Division.

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[4] J.W. Marangon Lima, R.F.A. Nunes, A.M. Leite da Silva, M.Th.Schilling, X. Vieira Filho, H.J. Chipp, "The Influence of Primary Energy Uncertainties on Probabilistic Load Flow", in the Proc. of the Third Int. Conference on PMAPS, London, UK, 3-5 July, 1991, IEE Pub. no. 338, pp. 305-310.

[5] X. Vieira Filho, J.W. Marangon Lima, A.M. Leite da Silva, M.Th. Schilling, V.L. Arienti, "Application of Probabilistic Load Flow in Operational Planning Studies: Current Status and Prospects", in the Proc. of the CIGRE Symp. on Electric Power Systems Reliability (EPSR), Montreal, 16-18 Sept., 1991, paper no. 5-01.

[6] A.C.G. Melo, M.V.F. Pereira, A.M. Leite da Silva, "Frequency and Duration Calculations in Composite Generation and Transmission Reliability Evaluation", paper no. 91 SM 440-8-PWRS, presented at the 1991 IEEE-PES Summer Meeting, San Diego, CA, July 28 - Aug. 1, 1991, (to be published in the IEEE Trans. on Power Systems).

[7] A.C.G. Melo, M.V.F. Pereira, A.M. Leite da Silva, "A Conditional Probability Approach to the Calculation of Frequency and Duration Indices in Composite Reliability Evaluation, paper submitted to the 1992 IEEE-PES Winter Meeting.

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[11] A.M. Leite da Silva, "Composite Generation and Transmission System Reliability - Adequacy and Security Assessment", June 1990, 86 pages.

[12] A.M. Leite da Silva, "Monte Carlo Simulation - Sequential Sampling", Unfinished Report, August 1990, 30 pages.

[13] A.M. Leite da Silva, J. Endrenyi, L. Wang, "A Framework for Integrated Adequacy and Security Assessment of Bulk Power Systems", in the Proceedings of the CIGRE Symposium on EPSR, Montreal, 16-18 September, 1991, paper no. 1A-03.

[14] A.M. Leite da Silva, J. Endrenyi, L. Wang, "Integrated Treatment of Adequacy and Security in Bulk Power System Reliability Evaluations", paper submitted to the 1992 IEEE-PES Winter Meeting/Transactions on Power Systems.

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APPENDIX: PAPER SUBMITTED FOR PRESENTATION AT THE 1992 IEEE-PES WINTER MEETING

INTEGRATED TREATMENT OF ADEQUACY AND SECURITY IN BULK POWER SYSTEM RELIABILITY EVALUATIONS

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ABSTRACT

Traditionally, bulk power system reliability evaluations have concentrated on the analysis of system adequacy, the ability to supply all loads within performance requirements. This paper deals with extending the scope of reliability evaluations to include a probabilistic assessment of system security, involving considerations of transient stability and the effects of cascading sequences. For each disturbance, first-impact states are defined which depend on the primary effect of the disturbance (static or dynamic problems, or no problem at all). Based on this analysis, a set of probabilistic indices is proposed to assess the security level of the pre-disturbance state. Next, the possibility and consequences of secondary effects such as cascading are considered, leading to another set of indices to measure load losses due to static and dynamic phenomena. The approach is illustrated in a numerical example.

Keywords: Composite power system, Adequacy, Security, Reliability, Reliability indices, Probabilistic transient stability, Bulk power system reliability.

INTRODUCTION

The reliability of bulk power systems (generation plus high-voltage transmission, also called composite systems) is an umbrella concept and not, per se, a measure of expected system performance. It is evaluated through indices relating to two of its attributes, adequacy and security [1, 2]. *Adequacy* is the ability of the system to supply the aggregate electric power and energy requirements of the customers, within component ratings and voltage limits, taking into account planned and unplanned component outages. *Security* is the ability of the system to withstand sudden disturbances such as the unanticipated losses of system components. Note that the "ability to withstand sudden disturbances" is interpreted as the ability to avoid system instability caused by such disturbances.

Clearly, security assessments involve transient stability evaluations, while adequacy assessments concentrate on the steady state conditions after a disturbance or incident, such as a component failure. When using the term security, however, it should be remembered that it is defined differently in other power system studies [3, 4].

Although much work has been carried out in the area of composite power system reliability, most investigations have only considered adequacy evaluations and are primarily planning-oriented [2, 5-10]. This calls for two extensions in the discussions, the integration of security aspects into the evaluations, and the study of operational reliability [11]. This paper is concerned with the former extension.

The application of probabilistic methods to the evaluation of system security, involving transient stability studies, has already received some attention [12-22]. These studies include developments in on-line dynamic security assessment [21, 22]. In present practice, however, stability evaluations are almost always performed by deterministic methods, even if they are combined with probabilistic adequacy studies [23]. There is still a lack of appropriate modeling techniques and of proper linkage between adequacy and security assessment, despite some recent work on this subject [24].

This paper is an extension of an earlier study by the authors reported in [25]. The discussion begins with a review of the concepts associated with power system disturbances, followed by an analysis of the states the system may pass through in the aftermath of a disturbance. Appropriate probabilistic indices are chosen and an approach for determining them is outlined. The discussion is completed with a numerical example.

POWER SYSTEM DISTURBANCES

General Concepts

Although it is very convenient to deal with power systems under steady state conditions, such conditions do not exist in the true sense. Random fluctuations in load are taking place all the time with subsequent adjustments of generation. At times, major disturbances occur, eg., a fault on a transmission line followed by its tripping, or a loss of a generating unit. In composite adequacy evaluation, the dynamics of the transition from one equilibrium state to another is neglected. Therefore, successful operation requires only that the new operating state does not violate any limits such as MW/Mvar generation, bus voltage magnitudes, line currents, etc.

Due to the transients after a disturbance, however, the new operating state may never be reached. The transients cause fluctuations in the system frequency, bus voltage magnitudes, etc. If these fluctuations exceed certain safe operating limits, loads may be disconnected in order to preserve a major portion of the system. Cascading events may appear and persist until the system completely separates or collapses. These effects are not accounted for in an adequacy study and must be captured in security evaluations.

Loss of stability can be the result of large or small disturbances. In the first category belong the generating capacity changes and line failures, especially the loss of highly loaded circuits and interconnections. On the other hand, random load fluctuations usually represent small disturbances. The traditional large disturbance stability problem is related to the short-term or transient period, which is usually limited to a few seconds following the disturbance. Small disturbances or small signal stability is concerned with the system response to small changes, and usually, the problem is one of ensuring sufficient damping of the system oscillations [26, 27]. In general, the magnitude of a small disturbance is such that the equations that describe the dynamics of the power system may be linearized for the purpose of the analysis. In this paper, if the system remains stable for the first few seconds following a severe disturbance, then it will be assumed stable for small perturbations about the post-fault system conditions. Therefore, only transient stability analysis will be considered.

The effect of a disturbance is manifest in the system state immediately after the disturbance, and also in the state the system reaches when all transients and cascading have died down and control and operator actions

have been taken. Before discussing the possible effects of disturbances, therefore, a relevant classification of system states is offered.

Classification of System States

Adequate (A) - All loads in the system are served with system components not stressed beyond their design ratings and bus voltages remaining within tolerances.

Inadequate (NA) - A state where some system elements are over-stressed (exposed to overloads) and/or some bus voltages are out of bounds.

Partially Adequate (PA) - After a limited amount of load is removed by some control/operator action, the remaining loads are served with the system components not stressed beyond their ratings and the bus voltages staying within tolerances.

Stable (ST) - Without further disturbance, the state remains in a steady-state condition.

Note: Clearly, states A, NA, and PA are all stable.

Unstable (NST) - The state will change its condition even if no further disturbance occurs.

Secure (S) - A stable state where no credible disturbance can result in system instability.

Not Secure (NS) - A stable state where some credible disturbances may result in system instability, initiating limited or uncontrolled cascading.

Note: If even one transition out of a state leads to instability, the state is not secure.

Marginally Adequate (MA) - An adequate state out of which some credible disturbances may transfer the system to a NA or NST state. Sometimes, this state is called the *alert* state [4, 5].

System Collapse - A state where the majority of loads are left without supply.

It should be observed that, in contrast with other states, the definitions of the S, NS and MA states are based on effects of disturbances yet to occur, rather than on the present condition of these states. This is further explained in the next section. The diagram in Figure 1 provides an overview of the relationship between the states defined above. Arrows indicate possible (future) transitions which make a state MA or NS or both.

The Impact of Disturbances

Figure 2 shows the possible sequences of events which a power system can experience after the occurrence of a disturbance. Note that steps of restoration which would complete the diagram are omitted for clarity. All the sequences shown are initiated by primary disturbance events such as faults, usually followed by a switching process. Examining the *first impact* of a disturbance, three situations may arise according to whether a static (state III) or dynamic (state IV) problem develops, or none of these problems exists (state II). The type of problem, if any, is determined through performing load flow analysis by solving a set of non-linear equations of the form

$$f(x_s) = 0 \qquad (1)$$

where x_s is the static state vector representing voltage magnitudes and angles, and performing transient stability analysis by solving a set of differential equations of the form

$$\dot{x}_d = g(x_d, t) \qquad (2)$$

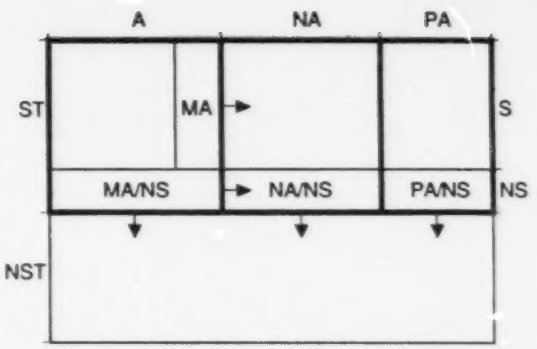

Figure 1. Classification of system states

where x_d is the dynamic state vector representing the state variables of generating units and $\dot{x}_d$ its time-derivative. Note that if the disturbance causes the system to pass through state III, then state I is MA. If it results in the system passing through state IV, state I is both MA and NS. The durations of states III and IV may vary from a few seconds (instability, generating capacity losses, heavy overloads) to minutes (small overloads).

The rest of the diagram in Figure 2 illustrates the possibilities of system behaviour after the immediate post-disturbance states III and IV. Several of these involve cascading sequences, limited or uncontrolled. Note that even static problems may trigger such sequences; in all cases they are due to a combination of protection system responses and operator action. The protection system and operator interventions are indicated in Figure 1 by crossed circles; actually, if cascading sets in, part of these actions will occur simultaneously with it. Unless the cascading becomes uncontrolled, operator actions culminate in taking corrective measures (indicated separately in the diagram) such as generation redispatch and load shedding. On the other hand, when cascading becomes uncontrolled, this will lead to system collapse (state IX).

In most cases, however, control and operator actions can achieve a stable A or PA state (states V, VI, VII and VIII). While some of these states may be adequate, it is unlikely that the generation dispatch in them is economical. If they are PA, their distinction may become important: it could help the planning engineer to better allocate reinforcements between transmission, generation, protection system, and emergency control scheme.

Most composite system reliability programs do not consider the dynamic impact of disturbances and the possibility of cascading; they cover only states II, III, and V [6, 7, 10, 28-30]. This is the traditional adequacy domain. All disturbances are considered small. Within this framework, Reference 10 pays particular attention to the MA states. Probabilistic transient stability studies [12-20, 28] do assess state IV and also model the protection system to some extent, but are not concerned with static problems. The need still persists to develop probabilistic techniques which can link the adequacy and security assessments of bulk power systems.

COMPONENT MODELING

In adequacy studies, the conventional two-state model shown in Figure 3a is generally used to represent transmission circuits. The failure rate λ and the repair rate μ in the model can be easily estimated from past performance. In security evaluations, a somewhat more sophisticated model is necessary to describe the failure-repair processes of transmission elements. While in adequacy studies the removal of a component has a unique effect on the system and the down-state in Figure 3a represents this condition, in security studies the effect is dependent on how fast the fault is cleared by the protection system, on the reliability of this system, and on the type of fault. Therefore, a different representation is needed.

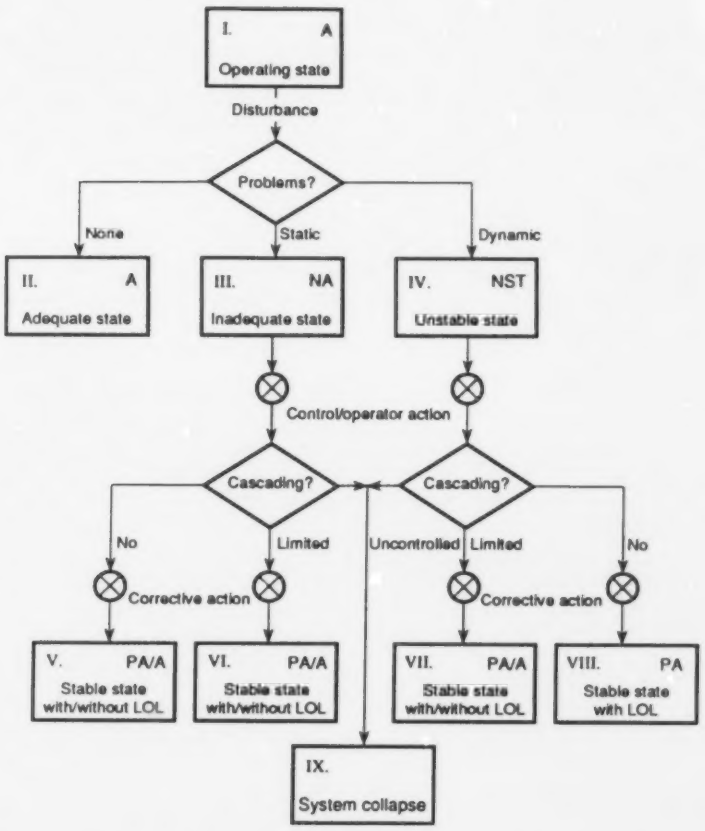

Figure 2. Possible consequences of disturbances in bulk power systems

The time in which the fault is removed is known as the *fault clearing time* CT. This duration can vary; its probability distribution function, $F_{CT}(t) = P[CT \leq t]$, depends upon the probability distributions of the operating times associated with the main and backup protection systems and with the circuit breakers protecting the transmission element in question [15, 28].

Another important parameter is the *critical clearing time* CCT. If the fault is cleared beyond this limit, the system will lose stability. The CCT is different for different disturbances, and can be determined by solving numerically equations 1 and 2, or by energy functions methods [17, 27]. The probability q of successfully clearing a fault is

$$q = F_{CT}(CCT) = P[CT \leq CCT] \qquad (3)$$

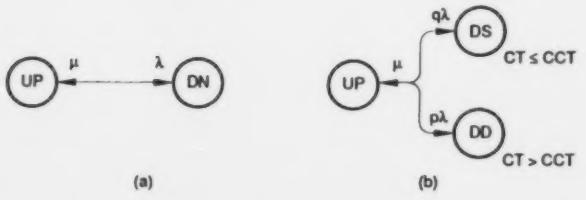

Figure 3. Transmission component outage models: (a) model for adequacy studies, (b) model accounting for static (DS) and dynamic (DD) effects

and the probability p of failing to do so is $p = 1 - q$. With this information, the model in Figure 3a can be adapted for security evaluations as shown in Figure 3b [25]. The two down-states in the modified model represents conditions where the loss of the component in question either does or does not endanger system stability.

As mentioned, a factor which has an effect on the outcome of security computations is the type of fault a transmission element may be experiencing. Different failure and repair rates can be estimated [31] for three-phase faults (λ_1,μ_1), single-phase-to-ground faults (λ_2,μ_2) and so on. Accordingly, the model in Figure 3b would have to be further split to accommodate all these possibilities. This is illustrated in Figure 4a.

To simplify matters, it will be assumed in this study that three-phase faults are the dominant sources of large disturbances. While three-phase faults occur far less frequently than, say, single-phase-to-ground faults, their effect is more severe and it is a common practice to use them as a basis for transient stability analysis. Consequently, the diagram in Figure 4a will be reduced to represent only two possibilities: three-phase faults (with subscript 1) and all other types together (subscript 0). Further, it will be assumed that the "0"-type faults have negligible dynamic impact, and thus $p_0 = 0$ and $q_0 = 1$. This leads to the model in Figure 4b, and in the following analysis, this will be the model used.

The models described can be extended, without difficulty, to accommodate all other failure types that are deemed important. Also the dynamic impact of generation and load disturbances can be assessed. These extensions, however, may result in significant increases in computing effort.

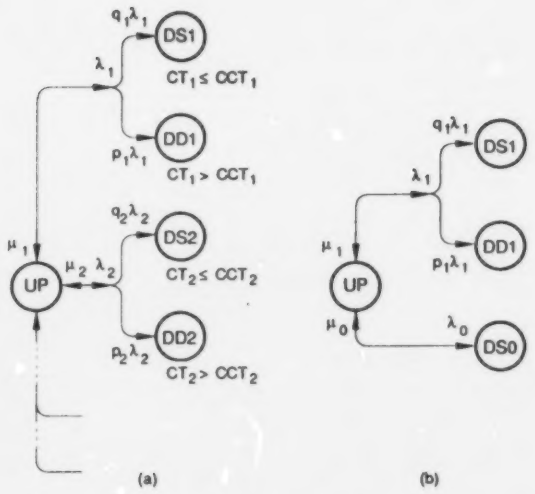

Figure 4. Models if several types of faults can occur: (a) general model, (b) only three-phase faults have dynamic effects

SYSTEM ANALYSIS

The same basic steps of analysis that are used in adequacy evaluations can also be used in studies covering both adequacy and security. These steps consist of state selection (by enumeration or simulation), state evaluation, and the computation of indices. The main difference is that now the states must be classified according to the possible outcomes shown in Figure 2.

The evaluation of each selected component outage state begins with determining whether, based on its *first impact*, it can be classified as adequate, stable but inadequate, or unstable (states II, III or IV in Figure 2). States II and III are distinguished through load flow analysis (Equation 1), and IV is identified through transient stability evaluation (Equation 2).

The next step is to determine whether the system stabilizes after the disturbance in question and if so, which of the states V to VIII (Figure 2) does it finally reach. The answer largely depends on the responses of the control and operator actions and, in particular, whether or not they trigger cascading. If there is no cascading, the final state after the disturbance can be easily identified (II, V or VIII in Figure 2) and the usual probability, frequency, etc. indices can be calculated. Although the same indices can be calculated if cascading occurs, the evaluation is more complex; this condition will be the subject of further research.

Returning to the first-impact states in Figure 2, one can compute the probabilities of encountering any of these states following a disturbance. Let $\lambda_{I,i}$, $i \in S_k$, represent the transition rates between a state of type I and a state i of type k (k = II, III, IV); S_k is the domain of type k states. The probability $P_{I,k}$ of encountering a state of type k after a disturbance is

$$P_{I,k} = \frac{\sum_i \lambda_{I,i}}{\lambda_I}, \quad i \in S_k, \quad k = II, III, IV \qquad (4)$$

where:

$$\lambda_I = \sum_i \lambda_{I,i}, \quad i \in (S_{II} \cup S_{III} \cup S_{IV})$$

Note that for, say, a transmission component with a three-phase failure rate λ_I, the associated transition rate $\lambda_{I,i}$, $i \in S_{IV}$, is equal to $p_I\lambda_I$. All these probabilities can be called *first-impact* indices. The last, $P_{I,IV}$, is the probability of instability or the degree of non-security for state I. If this is the base-case state, $P_{I,IV}$ is the probability of instability of the system.

The above equations are based on the assumption that the domain S_I contains only one state. The general form is derived in the Appendix. In the special case of Figure 4b where a single dominant failure mode is examined, Equation 4 for $P_{I,k}$ becomes much simpler. Assuming, for example, that the removal of a transmission line results in overload somewhere in the system,

$$P_{I,II} = 0, \quad P_{I,III} = \frac{q_I\lambda_I + \lambda_0}{\lambda_I + \lambda_0}, \quad P_{I,IV} = \frac{p_I\lambda_I}{\lambda_I + \lambda_0} \qquad (5)$$

In the given case, the probability $P_{I,IV}$ can also be calculated by the formula $p_I N_I/N$, where N_I is the number of times a three-phase fault occurs out of a total of N faults. In other words, the index $P_{I,IV}$ is the product of the probability that the protection system fails to remove a three-phase fault before the critical clearing time, and the probability of occurrence of the three-phase fault. This agrees with the definition of probability of instability in Reference 13. However, in practical situations where more than one transmission element is being considered, the assessment of index $P_{I,IV}$ by handling transition rates instead of counting the number of fault occurrences is preferable because external factors, such as the temporary disconnection of an element, may distort the counting process.

Another proposed first-impact index is the average time spent in states I, II, and III before the system reaches state IV; that is, the *mean time to instability*, MTTI. For the example in Figure 4b, the index MTTI can be evaluated by

$$MTTI = \frac{T_U + q_I T_I + T_0}{p_I N_I} \qquad (6)$$

where T_U is the total up-time, T_I the total duration of the three-phase outage, and T_0 the total duration of other outages, all during a given period of time. It is assumed that the times to instability are counted from the moment when the system leaves state I. Note that, if the fault is always eliminated before the critical clearing time ($p_I = 0$), or the number of three-phase faults is zero ($N_I = 0$), the index MTTI will tend to infinity; that is an unstable state is never reached. Considering all unstable states of type IV as absorbing [28,29], Equation 6 can be generalized for a system with any number of elements. This is also shown in the Appendix.

AN ILLUSTRATIVE EXAMPLE

System Characteristics

Figure 5 shows the configuration of a test system which basically consists of 5 buses and 3 transmissions lines. The input data for system reliability studies are provided in the following.

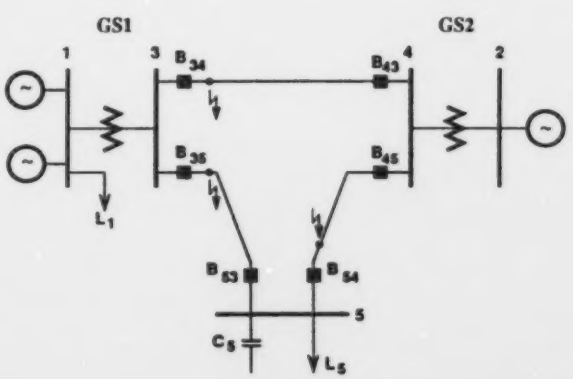

Figure 5. System configuration

Deterministic data: Generating station 1 has two 400 MW units, each with a transient reactance of 11% and an inertia constant of 15 s. Generating station 2 can be considered as an interconnection with a capacity of 600 MW, transient reactance of 5.5% and inertia constant of 3000 s. The classical machine model is used. The reactances are 4% for the transmission lines, 2% for the transformers and 440% for the shunt capacitor at bus 5. Bus loads are L_1 = 500MW/200 Mvar and L_5 = 800 MW/300 Mvar. A base power of 100 MVA is used.

Probabilistic data: All transformers, the shunt capacitor at bus 5 and the generation at bus 2 are considered 100% reliable. No uncertainties are assumed for the bus loads. For each transmission line, the average number of outages observed per year is 1 (three-phase) and 4 (others); the average repair time per outage per year is 20 h (three-phase) and 10 h (others). For the generators, the average number of outages observed per year is 3 and the average repair time per outage per year is 60 h. Assuming that the breakers are 100% reliable, the probability distribution function of the fault clearing time is given by [15, 28]:

$$F_{CT} = Q_m F_{CT/m}(t) + P_m F_{CT/b}(t) \qquad (7)$$

where Q_m and P_m are the probabilities of success and failure, respectively, of the main protection operation. These values will be initially assumed as 0.9 and 0.1, respectively. The functions F represent the probability distributions of the fault clearing times associated with the main (m) and back-up (b) protection. They are assumed normally distributed with expected values of 0.1 s (m) and 0.5 s (b), and a standard deviation of 10% of the mean. The basic system frequency is 60 Hz.

Criteria: Bus voltage magnitudes can range between 0.9 and 1.1 pu. Reactive capabilities are -300/+400 Mvar (bus 1) and -200/+300 Mvar (bus 2). The limit capacity of transmission elements is 700 MVA. The load shedding criterion is carried out in steps of 5% simultaneously on both loads. The three-phase fault locations are shown in the Figure 5. In case of failure of the main protection, the back-up protection will act as an emergency control scheme by isolating GS1 through opening circuit breakers B_{34} and B_{35}.

Results

In order to assess the *first impact* (FI) and *composite reliability* (CR) indices, the calculations were carried out in two steps. In the first, a set of algorithms (short circuit, AC load flow and transient stability) were used to classify each selected state. The CCT parameters calculated by the transient stability algorithm were: 0.18 s (line 3-4), 0.16 s (line 3-5) and 0.23 s (line 4-5). Only first order contingencies were considered (i.e. 11 states were analyzed) covering 99.86% of the probability space. The results of the state classification are shown in Table 1.

Table 1: State Classification

State No.	Failed Element	Component State (Figure 4b)	System States (Figure 2) FI	Final
1	none	-	I	-
2	G_1	DS0	III	V
3	L_{34}	DS0	II	-
4	L_{34}	DS1	II	-
5	L_{34}	DD1	IV	VII
6	L_{35}	DS0	III	V
7	L_{35}	DS1	III	V
8	L_{35}	DD1	IV	VII
9	L_{45}	DS0	III	V
10	L_{45}	DS1	III	V
11	L_{45}	DD1	IV	VII

In the second step, the FI and CR indices are evaluated. The CR indices are: LOLP = loss of load probability, LOLF = loss of load frequency, LOLD = loss of load duration, EENS = expected energy not supplied. They are separately assessed for static (S) and dynamic (D) problems. The EENS indices were calculated considering a year with 8760 hours. It was assumed that all cases with static problems (state III) would end up in state V; that is, the possibility of cascading was not considered. The cases with dynamic problems (state IV) were terminated after the action of the back-up protection. This calls for limited cascading and, therefore, the processes are assumed to end up in state VII; the possibility of uncontrolled cascading was discounted.

The results are summarized in Table 2. The FI indices indicate that this system will face post-disturbance inadequacy a significant number of times. The mean time to instability is 30 797 hours which is about 3½ years. Concerning the CR indices, the loss of load probability due to *static adequacy* problems, LOLP-S, is equal to 0.05196, while the loss of load probability due to *dynamic security* problems, LOLP-D, is equal to $0.648102\ 10^{-3}$. Clearly, static adequacy problems are predominant in this system. The total amount of load that has been shed in PA states V and VII are 805 MW and 1200 MW per year, respectively. In terms of expected power not supplied (EPNS), these figures become: EPNS-S = 143.18 MW and EPNS-D = 0.25924 MW.

Table 2: FI and CR Indices - Basic Results

FI Indices: $P_{I,II}$	$P_{I,III}$	$P_{I,IV}$	MTTI(h)
0.232405	0.753366	0.014229	30 797

CR Indices:	LOLP	LOLF (per yr)	LOLD (h)	EENS (MWh/yr)
(S)	$0.51960\ 10^{-1}$	$0.15949\ 10^{2}$	$0.28539\ 10^{2}$	$0.12543\ 10^{6}$
(D)	$0.64810\ 10^{-3}$	$0.29436\ 10^{0}$	$0.19287\ 10^{2}$	$0.22710\ 10^{4}$

Sensitivity Analysis

A series of tests were carried out to analyze the effects of variations of selected parameters on the reliability indices. Due to space limitations, only three such tests are presented.

Test 1: The sensitivities of the FI and CR indices to changes in the parameter P_m, the failure probability of the main protection, are examined. As can be seen in Figure 6, by increasing the chances of failure of the main protection, the index $P_{I,IV}$ increases, while $P_{I,II}$ and $P_{I,III}$ decrease. Thus, if P_m is higher, the system becomes less secure, as one would expect it. If the main protection system does not fail (P_m = 0), the index MTTI will tend to be very high, i.e. the system will hardly ever experience instability.

An analogous behaviour for CR indices is shown in Figure 7. As can be expected, by increasing the chances of failure of the main protection, the EENS due to static problems will slowly decrease while the EENS due to dynamic problems will increase. Since the amount of EENS is limited by the probability of a three-phase fault occurring, the curves actually flatten out as $P_m \rightarrow 1$. Other CR indices follow trajectories similar to those shown in Figure 7.

Test 2: The effects of varying the mean fault clearing times of the main protection, $E\{CT_m\}$, are shown in Figure 8. As expected, with increasing $E\{CT_m\}$, the system becomes less secure. Note that below a certain value of $E\{CT_m\}$, about 0.125 s, all indices run nearly constant with values depending only on the failure probability of the main protection, P_m = 0.1. This is because in that region all values of CT_m are likely to be below the critical clearing times, even when the standard deviation from the mean is 10%. Other CR indices follow trajectories similar to those shown in Figure 8.

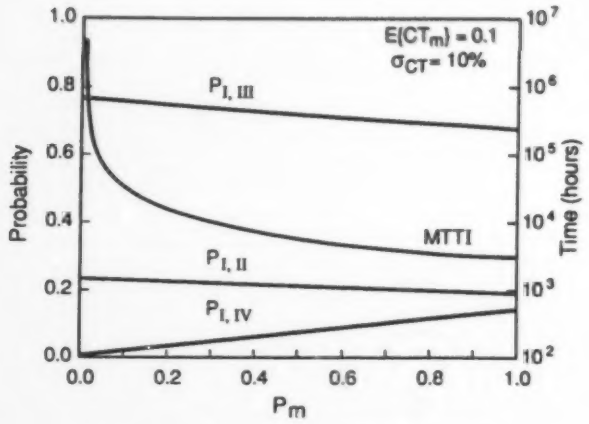

Figure 6. FI indices - sensitivity to changes in P_m

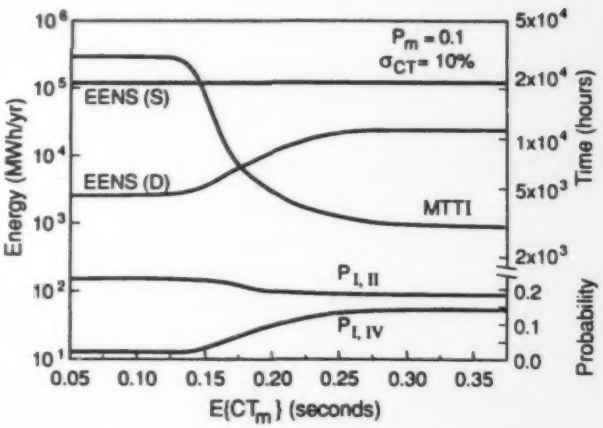

Figure 8. FI and CR indices - Sensitivity to changes in $E\{CT_m\}$

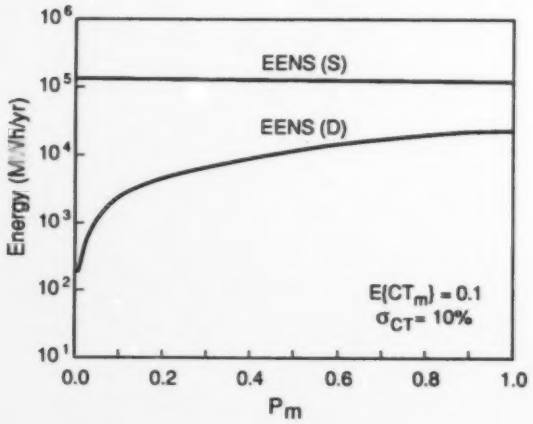

Figure 7. CR indices - sensitivity to changes in P_m

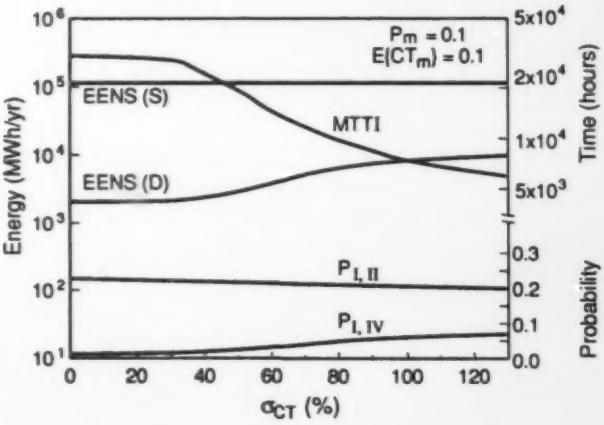

Figure 9. FI and CR indices - Sensitivity to changes in σ_{CT}

Test 3: The sensitivities of the FI and CR indices to changes in the standard deviation of the clearing times (σ_{CT}) are shown in Figure 9. Again, the system become less secure when this parameter increases. If there is no uncertainty in the fault clearing times ($\sigma_{CT} \rightarrow 0$) and also assuming that the main protection does not fail to operate ($Q_m \rightarrow 1$), the system with all components working becomes fully secure. This is true, of course, because the specified clearing times, all 0.1 s, are less than any of the calculated critical clearing times. Results for this case are shown in Table 3, and they represent the usual output obtained from an adequacy study. Clearly, the probability of instability is zero, the mean time to instability is infinity and all CR(D) indices are zero.

Table 3: FI and CR Indices - System Fully Secure

FI Indices:	$P_{I,II}$	$P_{I,III}$	$P_{I,IV}$	MTTI(h)
	0.237148	0.762852	0	∞

CR Indices:	LOLP	LOLF (per yr)	LOLD (h)	EENS (MWh/yr)
(S)	$0.52392\ 10^{-1}$	$0.16145\ 10^{2}$	$0.28427\ 10^{2}$	$0.12588\ 10^{6}$
(D)	0	0	0	0

OUTLOOK

This paper proposes a framework for combining the static and the dynamic effects of failures in power system reliability assessment. Successful assessment of dynamic problems depends, to a large extent, on being able to detect cascading sequences and evaluate their effects. This, in turn, requires accurate modeling of protection system actions, their failures, operator actions and possible human errors. These tasks are the subject of future research, some of which may be facilitated by the use of expert systems [32]. No lesser challenge is the development of feasible solution methods (already a problem when adequacy alone is considered) which will likely be based on sensitivity studies, advanced state selection techniques including state ranking algorithms and simulation methods including variance reduction techniques.

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APPENDIX
General Equations for The First Impact Indices

The diagram in Figure 10 describes the state space S of a Markov process where S is divided into the disjoint subsets S_α, S_β and S_γ. These domains represent a certain hierarchy: in order to reach S_γ from S_α, the process must pass through S_β. The state space S is a generalization of that in Figure 2, with S_α containing state I, S_β states II, III and IV, and S_γ the rest. The first task is to determine the probability of the system reaching a given state j in S_β, assuming that it left S_α. The second task is to determine the mean time spent in states I, II or III before the system reaches a state of type IV, i.e. an unstable state.

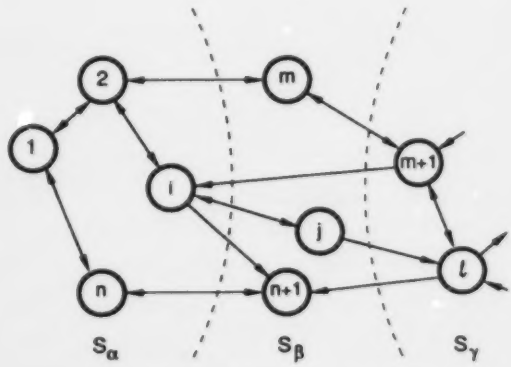

Figure 10. State space diagram $S = S_\alpha \cup S_\beta \cup S_\gamma$

First-Impact Probabilities

These probabilities can be fairly easily obtained if there is no return transition allowed from S_β to S_α, no communication among the states j in S_β, and if the transitions into the domain S_γ are neglected. Under these assumptions, the states in domain S_β become absorbing states, and their long-term probabilities p_β calculated for a given set of initial conditions with probabilities $p_\alpha(0)$ will provide the results sought in the first task above. The long-term solutions are needed because only as $t \to \infty$ will the system arrive in domain S_β with a probability approaching one. Note that the elements of vector p_β are $p_{m+1}, \ldots, p_n$ and those of $p_\alpha(0)$ are $p_1(0), \ldots, p_n(0)$.

In the given case, the transition intensity matrix A describing the possible transitions in S can be partitioned as follows:

$$A = \begin{array}{c} \begin{matrix} 1 \ldots\ n & n+1 \ldots\ m \end{matrix} \\ \left[\begin{array}{c|c} A_\alpha & A_\beta \\ \hline 0 & 0 \end{array}\right] \end{array} \qquad (8)$$

With these sub-matrices, the long-term probability vector p_β can be computed from the equation

$$p_\beta = -p_\alpha(0) A_\alpha^{-1} A_\beta \qquad (9)$$

Proof: In the general case, the time-dependent state probabilities of a Markov process are the solution of the matrix differential equation

$$\dot{p}(t) = p(t)\ A \qquad (10)$$

where $p(t)$ is the vector of the state probabilities at time t and $\dot{p}(t)$ is its time-derivative [28,29]. If discrete time is chosen, the equivalent equation will be

$$p(k+1) = p(k)\ \Pi \qquad (11)$$

where Π is the one-step transition probability matrix associated with the matrix A. With the assumptions stated above,

$$[p_\alpha(k+1) \mid p_\beta(k+1)] = [p_\alpha(k) \mid p_\beta(k)]\ \Pi \qquad (12)$$

where

$$\Pi = \left[\begin{array}{c|c} \Pi_\alpha & \Pi_\beta \\ \hline 0 & I \end{array}\right]$$

with I being the identity matrix. In the discrete-time case, matrix I replaces the corresponding 0 matrix because in an absorbing state the probability of staying there in the next step is 1, whereas the total rate of leaving it (required in A) is zero. Now, Equation 12 can be rewritten as:

$$\left.\begin{aligned} p_\alpha(k+1) &= p_\alpha(0)\ \Pi_\alpha^k \\ p_\beta(k+1) &= p_\alpha(0)\ \Sigma_\alpha(k)\ \Pi_\beta \end{aligned}\right\} \qquad (13)$$

where:

$$\Sigma_\alpha(k) = [I + \Pi_\alpha + \Pi_\alpha^2 + \ldots + \Pi_\alpha^k]$$

If $k \to \infty$, then $\Pi_\alpha^k \to 0$ and $\Sigma_\alpha(k) \to [I - \Pi_\alpha]^{-1}$. Therefore, for the stationary process,

$$\left.\begin{aligned} p_\alpha(\infty) &= 0 \\ p_\beta(\infty) &= p_\alpha(0)\ [I - \Pi_\alpha]^{-1}\ \Pi_\beta \end{aligned}\right\} \qquad (14)$$

Finally, the second Equation of (14) can be rewritten in terms of sub-matrices A_α and A_β, to provide Equation 9. This completes the proof.

If S_α contains only one state ($n = 1$, $p_\alpha(0) = 1$), the j-th element of p_β can be easily obtained from Equation 9:

$$p_j = \frac{\lambda_{1j}}{\lambda_1}, \qquad j = 2,\ldots,m \qquad (15)$$

where:

$$\lambda_1 = \sum_{j=2}^{m} \lambda_{1j}$$

Equation 15 is then applied to the states in Fig. 2 to provide Equation 4.

Mean Time to Instability

Turning now to the second task of generalizing Equation 6, Equation 9 is rewritten as:

$$p_\beta = T\ A_\beta \qquad (16)$$

where:

$$T = -p_\alpha(0)\ A_\alpha^{-1} \qquad (17)$$

The elements T(i), $i = 1,\ldots,n$ of vector T represent the weighted sum, for all possible initial conditions, of the mean times spent in state i ($i \in S_\alpha$) before arriving into a state in S_β [33]. That is, if the probability of the initial condition k, $k \in S_\alpha$, is $p_k(0)$ which is, of course, the k-th element of the vector $p_\alpha(0)$, then T(i), as given by the i-th equation of (17), can be interpreted as:

$$T(i) = \sum_{k=1}^{n} T(i|k)\ p_k(0) \qquad (18)$$

where $T(i|k)$ is the mean time in state i before crossing over to S_β under the assumption that the process started in state k.

When calculating the mean time to instability, one has to remember that only the type IV states in S_β represent instability. Therefore, for the purposes of the following step, it is expedient to redefine the domain S_β so as to contain only the states of type IV, and to transfer the type II and III states to the domain S_α. This domain now has n^* states instead of the original n; however, only for the original n states is $p_k(0)$ different from zero. Now, the index MTTI can be computed as:

$$MTTI = \sum_{i=1}^{n^*} T(i) \qquad (19)$$

This is a generalization of Equation 6.

BIOGRAPHIES

Armando M. Leite da Silva (S'77, M'78) was born in Rio de Janeiro (RJ), Brazil, in 1954. He received the B.Sc degree from the Catholic University of RJ (PUC-RJ) in 1975, the M.Sc degree from the Federal Univ. of RJ (COPPE-UFRJ) in 1977, and the Ph.D degree from the Univ. of Manchester (UMIST), England in 1980, all in Electrical Engineering. He is an Associate Professor in the Electrical Eng. Department at PUC-RJ. His primary research interests are the modelling, evaluation and application of probabilistic methods to power systems problems. He is currently with the Research Division of Ontario Hydro, Canada, on sabbatical leave from PUC-RJ.

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L. Wang (S'69, M'71, SM'84) received his undergraduate education in Taiwan, the M.Sc degree from the Univ. of Hawaii in 1963, and the Ph.D degree from Queen's University at Kingston, Canada in 1971, all in Electrical Engineering. He worked for Taiwan Power Company until 1961. Between 1963 and 1967 he taught Electrical Engineering at the Univ. of Hawaii and in Singapore. He joined the Research Division of Ontario Hydro in 1971 where he presently leads a small group of specialists working in power system reliability. He is a Registered Professional Engineer in the Province of Ontario.